

# Role of the synchronization point on boundary layer stabilization using porous coating

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## Abstract

The role of the synchronization point on stabilization of a Mach 5.92 flat-plate boundary layer using a porous coating is investigated in current paper by a combination of theoretical analysis and numerical simulations. The study is motivated by Fedorov et al.'s experimental and theoretical studies of the effect of an ultrasonically absorptive coating on hypersonic boundary-layer stability<sup>[1]</sup> and our previous numerical simulations on hypersonic boundary-layer receptivity<sup>[2]</sup>. The steady base flow is firstly simulated by solving compressible Navier-Stokes equations with a combination of a fifth-order shock-fitting method and a second-order total variation diminishing (TVD) scheme. The stability characteristics of the hypersonic boundary layer is analyzed by linear stability theory (LST). In receptivity simulations, periodic wall blowing-suction disturbance is introduced near the leading edge to excite boundary-layer waves. Then porous coating is used near the synchronization point to stabilize boundary-layer waves. The numerical results show that at the frequency of 100 kHz, porous coating destabilizes boundary-layer waves when it is located upstream of the synchronization point and stabilizes boundary-layer waves when it is located downstream of the synchronization point. These results indicate that the synchronization point plays an important role on the stabilization of the hypersonic boundary layer using porous coating. The most efficient way to stabilize boundary layer is to put porous coating downstream of the synchronization point. At the frequency of 150 kHz, the behavior of pressure perturbation is hard to interpret. The complex behavior of pressure perturbation may be caused by the coexistence of mode S, mode F, and continuous modes in the boundary layer, because the synchronization point is quite near to the blowing-suction actuator.

## 1 Introduction

Due to the fact that a turbulent boundary layer generates much higher shear stress and heat flux to the wall surface than a laminar boundary layer, the performance and thermal protection system of hypersonic transportation vehicles and re-entry vehicles are significantly affected by the laminar-turbulent transition of the boundary-layer flows over vehicle surfaces. Controlling transition and maintaining a laminar boundary layer as long as possible results in better fuel efficiency, lower drag and heat flux.

In order to predict and control boundary-layer transition, extensive studies have been carried out to reveal transition mechanisms<sup>[3, 4, 5, 6, 7, 8]</sup>. It is recognized that the transition process of a boundary-layer flow strongly depends on the amplitude level of environmental perturbations<sup>[9, 10]</sup>, which is schematically shown in Fig. 1. According to Reshotko's analysis<sup>[10]</sup>, only the first three paths had relevance to external flows. In an environment of small amplitude perturbations, transition of the boundary-layer flow over a smooth surface generally consists of the following three stages (path 1):

- Stage 1: Receptivity process during which small amplitude environmental disturbances enter the boundary layer and excite boundary-layer wave modes;
- Stage 2: Linear development or growth of unstable boundary-layer wave modes which can be predicted by solving the eigen-problem of the homogeneous linearized stability equations;

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- Stage 3: Boundary-layer transition caused by non-linear breakdown and three-dimensional effects when the unstable wave modes reach certain amplitudes.

For high amplitude perturbations, transient growth provided a higher initial amplitude to eigenmode growth (path 2). On non-smooth surface with stationary roughness elements, the strong transient growth of boundary-layer wave modes may directly lead to transition<sup>[9, 10]</sup> (path 3). All these transitions relating to high amplitude perturbations and transient growth are called bypass transition. In current paper, only small amplitude blowing-suction perturbation is considered. The undergoing path in this particular configuration is path 1. However the eigenmode growth of unstable boundary-layer wave is too weak to lead to transition.

In hypersonic boundary layers, two types of modes, mode S and mode F, are generally excited in receptivity process. These modes are given the names because they are tuned to slow and fast acoustic waves, respectively, in the limit of small Reynolds numbers. Mode F and mode S were first computed by Mack<sup>[11]</sup> for the compressible Blasius boundary layer. Mack found mode F to be stable, while mode S was found to be unstable and therefore more relevant in the transition process. Mode S constitutes both the first and second modes of Mack. The first mode is an inviscid instability wave, the counterpart of the Tollmien-Schlichting (T-S) waves at low-speed flows. Because compressible boundary-layer profiles contain an inflection point, the first mode begins to dominate with the Mach number increasing. The second mode belongs to a family of trapped acoustic modes, generally occurring for Mach numbers larger than 4. Once it is excited, the second mode is mode unstable than the first mode. According to the transition procedure, stabilizations of first mode and second mode are critical to transition control.

Techniques used to control laminar-turbulent transition can be divided into two types: passive control and active control. Passive control requires modification to the mean flow to inhibit the growth of boundary-layer modes, such as controlling mean surface temperature, modifying pressure gradient, and introducing localized steady surface suction. More passive control approaches were reviewed in Saric<sup>[12]</sup>. Active control aims to suppress the environmental disturbances. For example, the wave-like disturbances can be cancelled by introducing another wave of the same amplitude but opposite phase. Thomas<sup>[13]</sup> experimentally illustrated the concept of controlling boundary-layer transition by superimposing in the flow Tollmien-Schlichting waves that were of equal amplitude and antiphased to the disturbances. In all experiments, two electromagnetically excited vibrating ribbons were used to generate the instability waves in the flow, with the first ribbon working as an excitation source and the second one as an actuator. Both hot-wire surveys and flow visualization showed that it was possible to delay transition by appropriate adjustment of the phase and amplitude of the signal driving the second ribbon. Sturzebecher and Nitsche<sup>[14]</sup> showed that a sensor-actuator system combined with an adaptive control algorithm could successfully be used to achieve a powerful attenuation of naturally excited two-dimensional Tollmien-Schlichting (T-S) instabilities on an unswept wing. With a pure twodimensional control system, a local T-S wave was reduced in amplitude by about 90% . They also claimed that three-dimensional instabilities were successfully cancelled by means of a spanwise arrangement of sensors and actuators. Recently, Li and Gaster<sup>[15]</sup> numerically and experimentally investigated the control of spatially evolving three-dimensional instability waves in a flat-plate boundary layer with zero pressure gradient. They created the instability waves by computer-generated perturbations in a low-turbulence wind tunnel, then detected the oncoming disturbances and sent appropriate control signal to the downstream actuators to generate counteracting disturbances. They achieved good agreement between the experimental measurement and numerical predictions based on linear theory, which indicated that the growth of the three-dimensional instability waves could be inhibited by a simple control strategy.

Fedorov et al.<sup>[1]</sup> studied the effect of an ultrasonically absorptive coating (UAC) on hypersonic boundary-layer stability by experiment and theoretical analysis. Experiments were performed on a 7-degree half-angle sharp cone in a Mach 6 wind tunnel. Half-surface of the cone was solid and the other half-surface was covered by a porous coating. Both stability analysis and experimental results on two- and three-dimensional disturbances showed that the porous coating strongly stabilized the second mode and marginally destabilized the first mode. Since the first mode is spatially developed into the second mode downstream, destabilization of the first mode actually decreases the efficiency of the second mode stabilization. A straightforward way to avoid the destabilization of the first mode is to move the UAC coating further downstream to a location where the second mode is dominated. Based on previous numerical simulations on the receptivity of a Mach 8.0 flow to wall blowing-suction, Wang and Zhong<sup>[2]</sup> showed that the synchronization point of mode F and mode S played an important role in the excitation of mode S by wall blowing-suction. Mode S was strongly excited when the blowing-suction actuator was located upstream of the synchronization point. On the other hand, when the blowing-suction actuator was downstream of the synchronization point, there was a very weak excitation of mode S, despite the fact that the blowing-suction

actuator was still within the unstable region of mode S. Such results indicate that the synchronization point may be the perfect location to start UAC coating. The accuracy of our simulations has been validated by a concurrent theoretical study carried out by Tumin, Wang, and Zhong<sup>[16]</sup> to compare the theoretical and numerical results of receptivity coefficients and to analyze the receptivity characteristics. The perturbation flow field downstream of the blowing-suction actuator was decomposed into boundary-layer wave modes with the help of the biorthogonal eigenfunction system. It was found that there was a good agreement between normal-mode amplitudes calculated with the help of the theoretical receptivity model and those obtained by projecting the numerical results onto the normal modes.

In this paper, the role of the synchronization point on stabilization of a Mach 5.92 flat-plate boundary layer using a porous coating is investigated by a combination of theoretical analysis and numerical simulations. The free-stream flow conditions are the same as those of Maslov et al.'s leading-edge receptivity experiment<sup>[3]</sup>. The steady base flow is simulated by solving compressible Navier-Stokes equations with a combination of a fifth-order shock-fitting method and a second-order TVD scheme. The accuracy of the numerical steady base flow is validated by comparisons with the theoretical self-similar boundary-layer solution and Maslov et al.'s experimental measurements. With numerical base flow, the stability characteristics of the hypersonic boundary layer is analyzed by linear stability theory (LST). In stability simulations, periodic wall blowing-suction disturbance is introduced near the leading edge to excite boundary-layer waves. Then porous coating, modelled by pressure oscillation related wall blowing-suction as Fedorov et al. did, is used near the synchronization point to stabilize boundary-layer waves. Specifically, four cases of stability simulations are considered to investigate the role of the synchronization point on stabilization: case 1, without porous coating; case 2, porous coating located upstream of the synchronization point; case 3, porous coating located downstream of the synchronization point; case 4, porous coating located both upstream and downstream of the synchronization point.

## 2 Governing Equations and Numerical Methods

In the current numerical studies, a Mach 5.92 boundary layer over a flat plate as shown in Fig. 2 is considered. The flow is assumed to be thermally and calorically perfect. The governing equations for the simulation are the compressible Navier-Stokes equations in the conservative form, i.e.,

$$\frac{\partial \vec{U}^*}{\partial t^*} + \frac{\partial}{\partial x_1^*} (\vec{F}_{1i}^* + \vec{F}_{1v}^*) + \frac{\partial}{\partial x_2^*} (\vec{F}_{2i}^* + \vec{F}_{2v}^*) = 0 \quad (1)$$

where  $\vec{U}^*$  is a column vector containing the conservative variables of mass, momentum, and energy, i.e.,

$$\vec{U}^* = \{\rho^*, \rho^* u_1^*, \rho^* u_2^*, e^*\}^T \quad (2)$$

The superscript “\*” represents dimensional variables. The flux vectors in Eq. (1) are divided into their inviscid and viscous components, because the two components are discretized with different schemes. The components,  $\vec{F}_{1i}^*$  and  $\vec{F}_{2i}^*$ , are inviscid flux whereas  $\vec{F}_{1v}^*$  and  $\vec{F}_{2v}^*$  are viscous flux components. The flux components can be expressed as

$$\vec{F}_{ji}^* = \begin{bmatrix} \rho^* u_j^* \\ \rho^* u_1^* u_j^* + p^* \delta_{1j} \\ \rho^* u_2^* u_j^* + p^* \delta_{2j} \\ u_j^* (e^* + p^*) \end{bmatrix}, \quad \vec{F}_{jv}^* = \begin{bmatrix} 0 \\ -\tau_{1j}^* \\ -\tau_{2j}^* \\ -\tau_{1j}^* u_1^* - \tau_{2j}^* u_2^* - k^* \frac{\partial T^*}{\partial x_j^*} \end{bmatrix} \quad (3)$$

with  $j \in \{1, 2\}$ . In Eq. (3),  $\delta_{ij}$  ( $i = 1, 2$ ) is the Kronecker Delta function. In the perfect gas assumption, pressure and energy are given by

$$p^* = \rho^* R^* T^* \quad (4)$$

$$e^* = \rho^* c_v^* T^* + \frac{\rho^*}{2} (u_1^{*2} + u_2^{*2}) \quad (5)$$

where  $c_v^*$  is the specific heat at constant volume. For compressible Newtonian flow, the viscous stress tensor can be written as:

$$\tau_{ij}^* = \mu^* \left( \frac{\partial u_i^*}{\partial x_j^*} + \frac{\partial u_j^*}{\partial x_i^*} \right) - \frac{2}{3} \mu^* \left( \frac{\partial u_1^*}{\partial x_1^*} + \frac{\partial u_2^*}{\partial x_2^*} \right) \delta_{ij} \quad (6)$$

with  $i, j \in \{1, 2\}$ . In the simulation, the viscosity coefficient,  $\mu^*$ , and the heat conductivity coefficient,  $k^*$ , are calculated using the Sutherland's law together with a constant Prandtl number,  $Pr$ . They are both functions of temperature only.

$$\mu^* = \mu_r^* \left( \frac{T^*}{T_r^*} \right)^{3/2} \frac{T_r^* + T_s^*}{T^* + T_s^*} \quad (7)$$

$$k^* = \frac{\mu^* c_p^*}{Pr} \quad (8)$$

where  $\mu_r^* = 1.7894 \times 10^{-5}$  Ns/m<sup>2</sup>,  $T_r^* = 288.0$  K,  $T_s^* = 110.33$  K, and  $c_p^*$  is the specific heat at constant pressure.

In this paper, the dimensional flow variables are non-dimensionalized by free-stream parameters. Specifically, density, temperature, velocity, and pressure are non-dimensionalized by  $\rho_\infty^*$ ,  $T_\infty^*$ ,  $u_\infty^*$ , and  $\rho_\infty^* u_\infty^{*2}$ . Furthermore,  $x_1^*$  is non-dimensionalized by unit length in meter, whereas  $x_2^*$  is non-dimensionalized by the local boundary-layer thickness,  $\sqrt{\mu_\infty^* x_1^* / \rho_\infty^* u_\infty^*}$ . Referring to the coordinate system shown in Fig. 2,  $x_1^*$  and  $x_2^*$  are  $x^*$  and  $y^*$ , respectively. The two variables,  $u_1^*$  and  $u_2^*$ , are velocities in streamwise and wall-normal directions.

The fifth-order shock-fitting finite difference method of Zhong<sup>[17]</sup> is used to solve the governing equations in a domain bounded by the bow shock and the flat plate. In other words, the bow shock is treated as a boundary of the computational domain. The Rankine-Hugoniot relation across the shock and a characteristic compatibility relation coming from downstream flow field are combined to solve the flow variables behind the shock. The shock-fitting method makes it possible for the Navier-Stokes equations to be spatially discretized by high-order finite difference methods. Specifically, a fifth-order upwind scheme is applied to discretize the inviscid flux derivatives. Meanwhile, a sixth-order central scheme is used for the discretization of viscous flux derivatives. By using the shock-fitting method, the interaction between the bow shock and wall forcing induced perturbations is solved as a part of solutions with the position and velocity of the shock front being taken as dependent flow variables. A three-stage semi-implicit Runge Kutta method of Zhong et al.<sup>[18]</sup> is used for temporal integration. In the leading edge region, there exists a singular point at the tip of the flat plate, which introduces numerical instability if the fifth-order shock-fitting method is used to simulate the flow. Therefore, the computational domain for the shock-fitting simulation starts from a very short distance downstream of the leading edge. A second-order TVD scheme used by Zhong and Lee<sup>[19]</sup> is applied to simulate the steady base flow in a small region including the leading edge to supply inlet conditions for the shock-fitting simulation. In receptivity simulations, two-dimensional wall perturbations are introduced in a downstream region where the shock-fitting method is used.

The same numerical method has been used by Ma and Zhong in their receptivity studies of supersonic and hypersonic boundary layers over a flat plate and a sharp wedge to various free-stream disturbances<sup>[8, 20, 21]</sup>. The good agreement between numerical and LST results indicates that the fifth-order shock-fitting finite difference method is accurate to simulate the receptivity problems of high-speed boundary-layer flows. The numerical method has also been validated in the theoretical study and comparison with numerical simulation of Tumin, Wang, and Zhong<sup>[16]</sup>. The numerical perturbation field downstream of the blowing-suction actuator is decomposed into boundary-layer wave modes with the help of the biorthogonal eigenfunction system. The filtered-out amplitudes of mode S and mode F agree well with the corresponding theoretical solutions of linear receptivity problem.

### 3 Flow Conditions and Models of Blowing-Suction and Porous Coating

Free-stream flow conditions of the currently studied flow are the same as those of Maslov et al.'s experiment<sup>[3]</sup>, i.e.,

$$\begin{aligned} M_\infty &= 5.92, & T_\infty^* &= 48.69\text{K}, \\ p_\infty^* &= 742.76\text{Pa}, & Pr &= 0.72, \\ f^* &= 50\text{kHz}, & F &= 26.51 \times 10^{-6}, \\ Re_\infty^* &= 13 \times 10^6/\text{m} \end{aligned}$$

where  $Re_\infty^*$  is the unit Reynolds number defined as

$$Re_\infty^* = \rho_\infty^* u_\infty^* / \mu_\infty^* \quad (9)$$

The dimensional streamwise coordinate,  $x^*$  as shown in Fig. 2, can be converted to local Reynolds number by

$$Re_x = Re_\infty^* x^* \quad (10)$$

In LST studies of boundary-layer flows, the Reynolds number based on the local length scale of boundary-layer thickness,  $L^*$ , is generally used. They are expressed as

$$R = \frac{\rho_\infty^* u_\infty^* L^*}{\mu_\infty^*}, \quad L^* = \sqrt{\frac{\mu_\infty^* x^*}{\rho_\infty^* u_\infty^*}} \quad (11)$$

Hence, the relation between  $R$  and local Reynolds number  $Re_x$  is given by

$$R = \sqrt{Re_x} \quad (12)$$

For the simulation of steady base flow, the wall is adiabatic, and the physical boundary condition of velocity on the flat plate is the non-slip condition. When periodic blowing-suction disturbances and UAC are introduced, the isothermal temperature condition is applied on the wall. This temperature condition is a standard boundary condition for theoretical and numerical studies of high frequency disturbances. Meanwhile, non-slip condition is applied on the wall except the forcing region and UAC surface. In both steady and unsteady simulations, inlet conditions are specified, while high-order extrapolation is used for outlet conditions because the flow is hypersonic at the exit boundary except a small region near the flat plate. The flow variables behind the shock are solved by combining Rankine-Hugoniot relation across the shock and a characteristic compatibility relation coming from downstream flow field.

In this paper, a blowing-suction model similar to that of Eibler and Bestek<sup>[22]</sup> is used to excite boundary-layer wave modes. The mass flux oscillation on the flat plate within the blowing-suction region can be written as:

$$(\rho^* v^*)' = q_0^* \epsilon \beta(l) \sin \omega^* t^* \quad (13)$$

where  $q_0^*$  is a constant locally defined at the leading edge of the blowing-suction actuator, and  $\epsilon$  is a small dimensionless parameter representing the amplitude of the mass flux oscillation. The function  $\beta(l)$  is the profile function defined within the forcing region as

$$\beta(l) = \begin{cases} 20.25l^5 - 35.4375l^4 + 15.1875l^2 & \text{if } l \leq 1 \\ -20.25(2-l)^5 + 35.4375(2-l)^4 - 15.1875(2-l)^2 & \text{if } l \geq 1 \end{cases} \quad (14)$$

The variable  $l$  is a dimensionless coordinate defined within the blowing-suction region,

$$l = \frac{2(x^* - x_i^*)}{x_e^* - x_i^*} \quad (15)$$

where  $x_i^*$ ,  $x_e^*$  are the coordinates of the leading and trailing edges of the blowing-suction actuator. Due to the anti-symmetric property of the 5th-order-polynomial profile function within the blowing-suction region, the net mass flux introduced to the boundary layer is zero at any instant. In Eq. (13),  $\omega^*$  is the circular frequency of the blowing-suction disturbance, which is related to the frequency by

$$\omega^* = 2\pi f^* \quad (16)$$

The circular frequency,  $\omega^*$ , and the frequency,  $f^*$ , are non-dimensionalized according to

$$\omega = \frac{\omega^* L^*}{u_\infty^*} \quad (17)$$

$$F = \frac{2\pi f^* \mu_\infty^*}{\rho_\infty^* u_\infty^{*2}} = \frac{\omega^* \mu_\infty^*}{\rho_\infty^* u_\infty^{*2}} \quad (18)$$

With the definitions of Reynolds number  $R$  and the dimensionless frequency  $F$ , the dimensionless circular frequency can also be expressed as

$$\omega = RF \quad (19)$$

The current study focuses on linear responses of the boundary layer to wall blowing-suction. Therefore, the amplitude of mass flux oscillation is small enough to preserve the linear properties of the disturbances. The dimensionless amplitude coefficient,  $\epsilon$  in Eq. (13), is given as

$$\epsilon = 1.0 \times 10^{-5} \quad (20)$$

After the excitation of boundary-layer wave modes (mode F, mode S, and continuous modes), porous coating is used to try stabilizing unstable modes. In numerical simulations, the use of coating is represented by introducing another blowing-suction disturbance near the synchronization point. The model of the coating is the same as that of Fedorov et al. <sup>[1]</sup>, i.e., blowing-suction relating to the local pressure oscillation.

$$v' = A_y p' \quad (21)$$

The porous coating admittance,  $A_y$ , is defined as

$$A_y = -\frac{\phi}{Z_0} \tanh(\Lambda h) \quad (22)$$

In above equation,  $\phi$  is porosity,  $h$  is the porous-layer thickness non-dimensionalized by the local length scale of boundary layer thickness.

$$h = \frac{h^*}{L^*} = \frac{h^*}{\sqrt{\frac{\mu_\infty^* x^*}{\rho_\infty^* u_\infty^*}}} \quad (23)$$

Based on Allard and Champoux's theoretical analysis <sup>[23]</sup>, the empirical equations for porous coating characteristic impedance ( $Z_0$ ) and propagation constant ( $\Lambda$ ) are

$$Z_0 = \frac{\rho_w}{M_e} \sqrt{\frac{T_w \tilde{\rho}}{\tilde{C}}} \quad (24)$$

$$\Lambda = \frac{i\omega M_e}{\sqrt{T_w}} \sqrt{\tilde{\rho} \tilde{C}} \quad (25)$$

where  $\rho_w$  and  $T_w$  are the local dimensionless density and temperature on coating. The edge Mach number ( $M_e$ ) is defined right after the shock wave. The dynamic density ( $\tilde{\rho}$ ) and bulk modulus ( $\tilde{C}$ ) are calculated using the following two equations.

$$\tilde{\rho} = a_\infty \left[ 1 + \frac{g(\lambda_1)}{\lambda_1} \right] \quad (26)$$

$$\tilde{C} = \gamma - \frac{\gamma - 1}{1 + \frac{g(\lambda_2)}{\lambda_2}} \quad (27)$$

where  $\lambda_1$ ,  $\lambda_2$ , and the function  $g(\lambda)$  are defined as

$$g(\lambda) = \sqrt{1 + \frac{4a_\infty \mu_w^* \lambda}{\sigma^* \phi r_p^{*2}}} \quad (28)$$

$$\lambda_1 = \frac{ia_\infty \rho_w^* \omega^*}{\phi \sigma^*} \quad (29)$$

$$\lambda_2 = 4\text{Pr}\lambda_1 \quad (30)$$

where  $a_\infty$  is the tortuosity,  $\sigma^*$  is the flow resistivity, and  $r_p^*$  is the characteristic pore size. These parameters are determined by the porous coating. In porous coating model, the local dimensionless variables ( $v$ ,  $p$ ,  $T_w$ ,  $\rho_w$ ,  $\omega$ ) are non-dimensionalized by the corresponding variables right after the shock wave ( $u_e^*$ ,  $\rho_e^* u_e^{*2}$ ,  $T_e^*$ ,  $\rho_e^*$ ,  $\sqrt{\mu_e^* x^* / \rho_e^* u_e^* / u_e^*}$ ).

According to the definitions of characteristic impedance and propagation constant, the porous coating admittance is generally a complex number. And the pressure perturbation is also a complex number. However, only the real part of velocity perturbation can be imposed on the coating in numerical simulations. In this way, the velocity perturbation is written relating to the real part of pressure perturbation ( $p(t^*)$ ) as

$$v(t^*) = \text{Real}(A_y)p(t^*) - \text{Imag}(A_y)p(t^* - \frac{\pi}{2\omega^*}) \quad (31)$$

where “Real( $A_y$ )” and “Imag( $A_y$ )” represent the real part and imaginary part of porous coating admittance, respectively.

## 4 Results and Discussions

### 4.1 Steady base flow

The steady base flow over the flat plate is simulated by solving the two-dimensional compressible Navier-Stokes equations with a combination of a fifth-order shock-fitting finite difference method and a second-order TVD scheme. In the leading edge region, there exists a singular point at the tip of the flat plate, which will introduce numerical instability if the fifth-order shock-fitting method is used to simulate the flow. Therefore, a second-order TVD scheme used by Zhong and Lee<sup>[19]</sup> is applied to simulate the steady base flow in a small region including the leading edge. The computational domain for the fifth-order shock-fitting method starts at  $x^* = 0.0025$  m and ends at  $x^* = 0.879$  m, corresponding to  $R = 180.28$  and  $R = 3380.38$ , respectively. In actual shock-fitting simulations, the computational domain is divided into 19 zones with a total of 3746 grid points in streamwise direction. The number of grid points in wall-normal direction is 121 upstream of the position of  $x^* = 0.309$  m and 176 downstream of that position. Forty-one points are used in the overlapped region between two neighboring zones, which is proved to be sufficient to make the solution accurate and smooth within the whole domain. An exponential stretching function is used in the wall-normal direction to cluster more points inside the boundary layer. On the other hand, the grid points are uniformly distributed in streamwise direction. The spatial convergence of the results based on this grid structure has been evaluated by grid refinement studies to ensure the grid independence of the fifth-order shock-fitting simulations.

For shock-fitting simulation in the first zone, the inlet conditions are obtained from the results of the second-order TVD shock-capturing scheme which is used to simulate the steady base flow in a small region including the leading edge. For shock-fitting simulations in other zones, inlet conditions are interpolated from the results of the previous zone. The computational domain for the second-order TVD scheme starts at  $x^* = -0.0005$  m and ends at  $x^* = 0.0035$  m. Three sets of grid structures are used to check the grid independence of the numerical results. Figure 3 compares the density contours near the leading edge simulated by second-order TVD scheme based on three sets of grid structures. It shows that the contours on  $201 \times 176$  mesh agree well with those on  $241 \times 181$  mesh whereas they have significant discrepancies with the contours on  $161 \times 101$  mesh. This figure indicates that the grid structure of  $201 \times 176$  is fine enough to ensure the grid independence of numerical results. However, the grid structure of  $161 \times 101$  is too coarse to achieve spatially converged numerical results. Similar conclusion can be drawn in Fig. 4, where the density distributions in wall-normal direction are compared. In this figure, the density distributions are evaluated at  $x^* = 0.0025$  m, the initial location of the first zone.

In order to validate the combination of fifth-order shock-fitting method and second-order TVD scheme, Figs. 5 and 6 plot density and wall-normal velocity contours near the leading edge of the steady base flow obtained by the combination of second-order TVD scheme and fifth-order shock-fitting method. The flow field including the leading edge is simulated by the TVD scheme, while the flow field after  $x^* = 0.0025$  m is simulated by the shock-fitting method. These figures show that density and wall-normal velocity contours calculated by the two methods have a good agreement near the upstream edge of the overlapped region, which indicates that the TVD solutions are accurate enough to be used as inlet conditions for the fifth-order shock-fitting simulation in the first zone. The small discrepancies of the contours near the bow shock are caused by viscous effect. Due to viscosity, the bow shock has a finite thickness for TVD simulation, while it is assumed to be infinitely thin for the shock-fitting simulation. The combination of fifth-order shock-fitting method and second-order TVD scheme has also been validated in cases of supersonic and hypersonic steady base flows by Ma & Zhong<sup>[24]</sup> and Wang & Zhong<sup>[2]</sup>.

To validate the accuracy of numerical simulation, the steady base flow simulated by the fifth-order shock-fitting method is compared with Maslov et al.'s experimental measurements and the theoretical self-similar boundary-layer solution. According to Maslov et al.'s experiments, effect of the viscous-inviscid interaction is strong within the region of  $0.2 \text{ mm} < x^* < 33 \text{ mm}$ . Figure 7 shows that pressure has large gradients upstream of the location of  $x^* = 33 \text{ mm}$ , which is consistent with the experimental results. Figure 8 shows the distributions of dimensionless streamwise velocity across the boundary layer at three different locations. It shows that the thickness of the boundary layer increases with the location shifting downstream. At  $x^* = 96 \text{ mm}$ ,  $121 \text{ mm}$ , and  $138 \text{ mm}$ , the thicknesses of the boundary layer are approximately equal to  $1.84 \text{ mm}$ ,  $2.04 \text{ mm}$ , and  $2.20 \text{ mm}$ , which has a good agreement with Maslov et al.'s experimental measurements of about  $1.8 \text{ mm}$ ,  $2.0 \text{ mm}$ , and  $2.2 \text{ mm}$ , respectively.

Figures 9 and 10 compare normalized Mach number  $M/M_\infty$  and dimensionless streamwise velocity  $u^*/u_\infty^*$  distributions across the boundary layer at three different locations of  $x^* = 96 \text{ mm}$ ,  $121 \text{ mm}$ , and  $138 \text{ mm}$ . In these figures,  $\eta$  is defined as  $\eta = y^*/L^*$ . The solid lines represent distributions of  $M/M_\infty$  and  $u^*/u_\infty^*$  obtained by solving the compressible boundary-layer equations. Due to the fact that the theoretical solution of boundary-layer equations is self-similar, the distributions of  $M/M_\infty$  and  $u^*/u_\infty^*$  at different locations are exactly the same. The unfilled symbols represent experimental results of Maslov et al., whereas the other three lines stand for numerical simulation results. The good agreement between the simulation results indicate that an approximate gradientless flow is obtained over the flat plate, which is evaluated in Maslov et al.'s paper by comparing the experimental results. The approximate gradientless flow is clearly shown in Fig. 7. Figures 9 and 10 show that the numerical results agree well with the experimental results and the boundary-layer solution near the plate. However, in the region of  $\eta > 5$ , the numerical results have a better agreement with the experimental results. The difference between the numerical results and the boundary-layer solution is mainly caused by viscous-inviscid interaction, because the effect of the bow shock is neglected in the calculation of the compressible boundary-layer equations. The analysis of Figs. 8, 9, and 10 indicate that the hypersonic steady base flow considered in the current study is accurately simulated by the fifth-order shock-fitting method.

#### 4.2 Stability characteristics of boundary-layer wave modes

The stability characteristics of boundary-layer wave modes of the Mach 5.92 flow is studied by LST based on a multi-domain spectral method of Malik <sup>[25]</sup>. The velocity, pressure, and temperature disturbances are represented by harmonic waves of the form

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \\ \tilde{p} \\ \tilde{T} \end{pmatrix} = \begin{pmatrix} \hat{u}(y) \\ \hat{v}(y) \\ \hat{w}(y) \\ \hat{p}(y) \\ \hat{T}(y) \end{pmatrix} e^{i(\alpha_c x + \beta_c z - \omega_c t)} \quad (32)$$

The parameters of  $\alpha_c$  and  $\beta_c$  are wave number components in streamwise and spanwise directions, and  $\omega_c$  is the circular frequency. For two-dimensional wall perturbations,  $\beta_c = 0$ . Substituting disturbances of Eq. (32) and the steady base flow into the compressible linearized Navier-Stokes equations, an ordinary-differential-equation system is achieved, i.e.,

$$\left( A \frac{d^2}{dy^2} + B \frac{d}{dy} + C \right) \phi = 0 \quad (33)$$

where  $\phi$  is the disturbance vector defined by  $\{\hat{u}, \hat{v}, \hat{p}, \hat{T}, \hat{w}\}^T$ . The coefficient matrices of A, B, and C are given in Malik's paper <sup>[25]</sup>. In spatial stability analysis, the two parameters,  $\omega_c$  and  $\beta_c$ , are specified as real numbers. The streamwise wave number,  $\alpha_c$ , is a complex number and solved as the eigenvalue of the ordinary-differential-equation system. The complex wave number  $\alpha_c$  can be expressed as

$$\alpha_c = \alpha_r + i\alpha_i \quad (34)$$

where  $\alpha_i$  is the local growth rate. A boundary-layer wave mode is unstable when  $\alpha_i < 0$  whereas it is stable when  $\alpha_i > 0$ . The points  $\alpha_i = 0$  are called the neutral points of the boundary-layer wave mode. A wave mode is generally unstable only in certain domains bounded by the neutral points. The real part,  $\alpha_r$ , is the local wave number which

can be used to define the local phase velocity:

$$a = \frac{\omega_c}{\alpha_r} \quad (35)$$

Both the wave number and the phase velocity can be used to identify the boundary-layer wave mode.

Figure 11 shows eigenvalue spectra of wave number  $\alpha$  of the boundary-layer wave modes for a case with a frequency of  $f^* = 100$  kHz at  $x^* = 0.189$  m ( $F = 53.03 \times 10^{-6}$ ,  $R = 1567.48$ ). The figure shows the wave spectra corresponding to fast acoustic mode, entropy and vorticity mode, and slow acoustic mode. The two discrete modes marked by circles are mode F and mode S. Mode F is a stable mode, which was called Mode I by Ma and Zhong [20, 21]. This mode originates from the fast acoustic spectrum on the left side of the figure and passes the entropy and vorticity spectra at the center as dimensionless frequency increases. Mode S originates from the slow acoustic spectrum on the right side of the figure. It becomes unstable in certain range of dimensionless frequency. The figure also shows that mode S at the frequency of  $f^* = 100$  kHz is unstable at  $x^* = 0.189$  m with  $\alpha_i < 0$ .

Figure 12 shows the dimensionless phase velocities of boundary-layer wave modes at two locations of  $x^* = 0.159$  m and  $x^* = 0.189$  m as a function of the dimensionless circular frequency. The three horizontal dashed lines represent the dimensionless phase velocities of fast acoustic mode ( $a = 1 + M_\infty^{-1}$ ), entropy and vorticity modes ( $a = 1$ ), and slow acoustic mode ( $a = 1 - M_\infty^{-1}$ ), respectively. The excellent agreement of the phase velocities at different locations indicates that the phase velocity is approximately a function of dimensionless circular frequency only. The figure clearly shows that mode F originates from fast acoustic mode. As  $\omega$  increases, the phase velocity of mode F decreases. When mode F passes the continuous entropy and vorticity modes near  $\omega = 0.1$ , there exists a jump of phase velocity, which is consistent with the theoretical analysis of Fedorov and Khokhlov [26]. On the other hand, mode S originates from slow acoustic mode. The figure also shows that mode S synchronizes with mode F at the point of  $\omega_s = 0.11563$  and  $a_s = 0.93076$ .

Although Fig. 12 shows that the synchronization point has a constant value of dimensionless circular frequency ( $\omega_s = 0.11563$ ), the dimensional location of the synchronization point,  $x_{sn}^*$ , are different for different dimensionless frequencies. The synchronization location in  $x^*$  coordinate for a given dimensionless frequency can be calculated using the following formula

$$x_s^* = \frac{(\omega_s/F)^2}{Re_\infty^*} \quad (36)$$

This equation indicates that the synchronization point moves upstream with the dimensionless frequency increasing.

### 4.3 Effect of porous coating on pressure perturbation

In current study, the same porous coating parameters as those of Fedorov et al. [1] are used, i.e.,

$$\begin{aligned} \phi &= 0.75, & h^* &= 0.75\text{mm}, \\ a_\infty &= 1, & d^* &= 30\mu\text{m}, \\ \sigma^* &= 1.66 \times 10^5 \text{kg}(\text{m}^3\text{s}^{-1}), & \gamma &= 1.4 \end{aligned}$$

where  $d^*$  is the fiber diameter, which is related to the characteristic pore size as follows:

$$r_p^* = \frac{\pi d^*}{2 - 3\phi + \phi^2} \quad (37)$$

For stability simulations, single-frequency blowing-suction disturbance is used to excite boundary-layer wave modes. After the excitation of unstable wave modes, porous coating is used near the synchronization point to stabilize boundary-layer waves. Specifically, four cases of stability simulations are considered to investigate the role of the synchronization point on stabilization: case 1, without porous coating; case 2, porous coating located upstream of the synchronization point; case 3, porous coating located downstream of the synchronization point; case 4, porous coating located both upstream and downstream of the synchronization point. The locations of porous coating for the four cases are schematically shown in Fig. 13.

Blowing-suction disturbances at two frequencies are considered in this section, i.e.,

- $f^* = 100$  kHz,  $F = 53.03 \times 10^{-6}$
- $f^* = 150$  kHz,  $F = 79.54 \times 10^{-6}$ .

According to Eq. (36), the synchronization points at these two frequencies are located at  $x^* = 0.33184$  m and  $x^* = 0.14748$  m, respectively. Blowing-suction disturbances are introduced in the region of  $33 \text{ mm} \leq x^* \leq 37 \text{ mm}$ , upstream of the synchronization point.

Figures 14 and 15 show amplitude distributions of pressure perturbation along the flat plate excited by single-frequency blowing-suction. It is noticed that significant increase in pressure perturbation amplitude happens near the corresponding synchronization point. In order to check the properties of the unstable modes, a local wave number ( $\alpha_r$ ) and a local growth rate ( $\alpha_i$ ) of the perturbation at the frequency of 100 kHz is calculated by

$$\alpha_r = L^* \frac{d\phi'}{dx^*} \quad (38)$$

$$\alpha_n = -\frac{L^*}{|p'|} \frac{d|p'|}{dx^*} \quad (39)$$

where  $L^*$  is the length scale of local boundary layer thickness as defined by Eq. (11).  $|p'|$  and  $\phi'$  are the perturbation amplitude and phase angle, respectively. The parameters  $\alpha_r$  and  $\alpha_i$  will represent the true wave number and growth rate only if the disturbance is dominated by a single wave mode. Otherwise, the disturbance needs to be decomposed in order to check properties of a specific mode. For example, Tumin, Wang, and Zhong<sup>[16]</sup> decomposed the disturbance at a location just downstream of the blowing-suction actuator with a biorthogonal eigenfunction system, where mode F, mode S, and acoustic modes coexisted and none of them was dominant.

Since our focus is on numerical simulation results, we only consider the later stage where mode S is the dominant mode in the boundary layer. In this case, Eqs. (38) and (39) can be used to check the properties of the unstable mode S. Figure 16 compares numerical growth rate and LST growth rate of the pressure perturbation at the frequency of 100 kHz. The growth rate of the simulation result has a good agreement with the LST growth rate in the region from  $\omega = 0.11$  to  $\omega = 0.13$ . When  $\omega$  is larger than 0.13, the growth rates from simulation are larger than those from LST, i.e., mode S obtained by the numerical simulation becomes more stable than that predicted by LST. Figure 17 compares wave number of the perturbation obtained from simulation at the frequency of 100 kHz with that of LST. The good agreement of the two sets of wave numbers in downstream region of  $\omega > 0.09$  confirms that mode S is the dominant mode in the boundary layer. The agreement in  $\alpha_r$  is not so good in early region because mode F, mode S, and acoustic modes coexist in the boundary layer. In order to check the properties of mode S, the perturbation in early region needs to be decomposed into individual wave components.

At the frequency of 100 kHz, the locations of porous coating in cases 2, 3, and 4 are as follows:  $0.2835 \text{ m} \leq x^* \leq 0.3048 \text{ m}$ ,  $0.3665 \text{ m} \leq x^* \leq 0.4020 \text{ m}$ ,  $0.3165 \text{ m} \leq x^* \leq 0.3470 \text{ m}$ . Figure 18 compares amplitude distributions of pressure perturbation along the flat plate for the four cases at this frequency. This figure shows that pressure perturbation decreases for all the four cases within the porous coating region. However, pressure perturbation of case 2 increases much faster than other cases and has the strongest amplitude further downstream. Pressure perturbation of case 4 has a larger amplitude than that of case 3 whereas it has a smaller amplitude than that of case 1. With the discussions on Figs. 16 and 17, it is already demonstrated that the increase of pressure perturbation amplitude is caused by the instability of mode S. The fact that pressure perturbation of case 2 has the largest amplitude indicates that mode S has the strongest amplitude for this case. In case 2, porous coating is located upstream of the synchronization point, where mode S is also called Mack first mode. In cases 3 and 4, porous coating is mainly located downstream of the synchronization point, where mode S is also called Mack second mode. The difference of pressure perturbation amplitudes among the four cases can be explained by Fedorov et al.'s theoretical and experimental results<sup>[1]</sup>: porous coating stabilizes the second mode whereas destabilizes the first mode. The numerical simulation results show that the synchronization point plays an important role on the stabilization of the hypersonic boundary layer using porous coating. The most efficient way to stabilize boundary layer is to put porous coating downstream of the synchronization point.

At the frequency of 150 kHz, the locations of porous coating move upstream due to the shift of synchronization point. In cases 2, 3, and 4, they are as follows:  $0.0735 \text{ m} \leq x^* \leq 0.1248 \text{ m}$ ,  $0.1935 \text{ m} \leq x^* \leq 0.2448 \text{ m}$ ,  $0.1323 \text{ m} \leq x^* \leq 0.1656 \text{ m}$ . Figure 19 compares amplitude distributions of pressure perturbation along the flat plate for the four cases at this frequency. This figure also shows that pressure perturbation decreases for all the four cases within the porous coating region. However, pressure perturbations of cases 2, 3, and 4 all have smaller amplitudes than that of case 1. Compared with the results shown in Fig. 18, the behavior of pressure perturbation is hard to interpret. The complex behavior of pressure perturbation may be caused by the coexistence of mode S, mode F,

and continuous modes in the boundary layer, because the synchronization point is quite near to the blowing-suction actuator.

## 5 Summary

The role of the synchronization point on stabilization of a Mach 5.92 flat-plate boundary layer using a porous coating is investigated by a combination of theoretical analysis and numerical simulations. The free-stream flow conditions are the same as those of Maslov et al.'s leading-edge receptivity experiment<sup>[3]</sup>. The steady base flow is firstly simulated by solving compressible Navier-Stokes equations with a combination of a fifth-order shock-fitting method and a second-order total variation diminishing (TVD) scheme. The accuracy of the numerical steady base flow is validated by comparisons with the theoretical self-similar boundary-layer solution and Maslov et al.'s experimental measurements<sup>[3]</sup>. The stability characteristics of the hypersonic boundary layer is analyzed by linear stability theory (LST). In receptivity simulations, periodic wall blowing-suction disturbance is introduced near the leading edge to excite boundary-layer waves. Then porous coating is used near the synchronization point to stabilize boundary-layer waves. Specifically, four cases of stability simulations are considered to investigate the role of the synchronization point on stabilization: case 1, without porous coating; case 2, porous coating located upstream of the synchronization point; case 3, porous coating located downstream of the synchronization point; case 4, porous coating located both upstream and downstream of the synchronization point. The numerical results show that at the frequency of 100 kHz, porous coating destabilizes boundary-layer waves when it is located upstream of the synchronization point and stabilizes boundary-layer waves when it is located downstream of the synchronization point. These results indicate that the synchronization point plays an important role on the stabilization of the hypersonic boundary layer using porous coating. The most efficient way to stabilize boundary layer is to put porous coating downstream of the synchronization point. At the frequency of 150 kHz, the behavior of pressure perturbation is hard to interpret. The complex behavior of pressure perturbation may be caused by the coexistence of mode S, mode F, and continuous modes in the boundary layer, because the synchronization point is quite near to the blowing-suction actuator.

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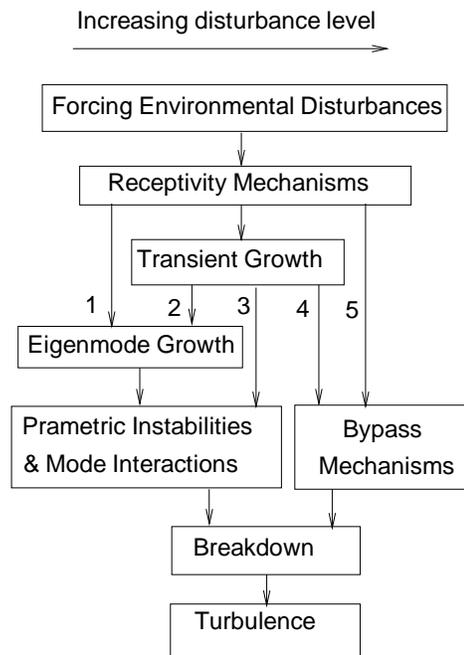


Figure 1: Paths of the transition process in boundary layers respecting to disturbance amplitude level.

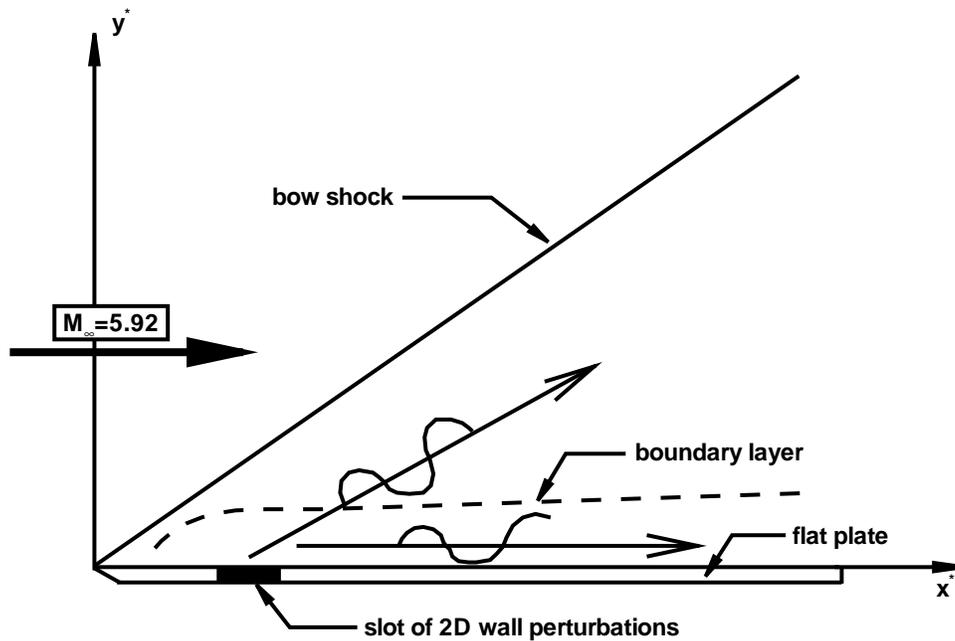


Figure 2: A schematic of the receptivity of a Mach 5.92 flow over a flat plate to periodic two-dimensional wall perturbations.

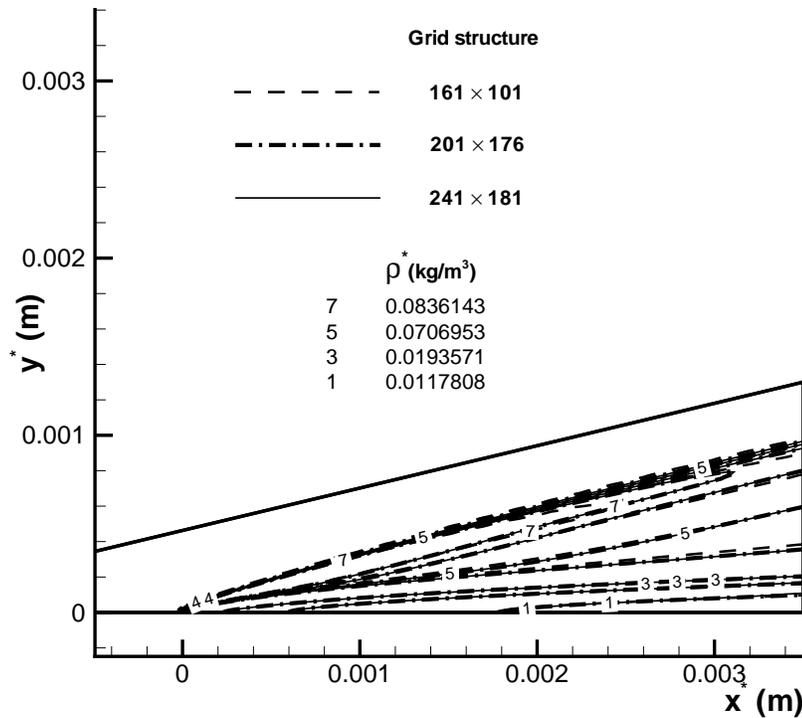


Figure 3: Comparison of density contours near the leading edge simulated by second-order TVD scheme based on three sets of grid structures.

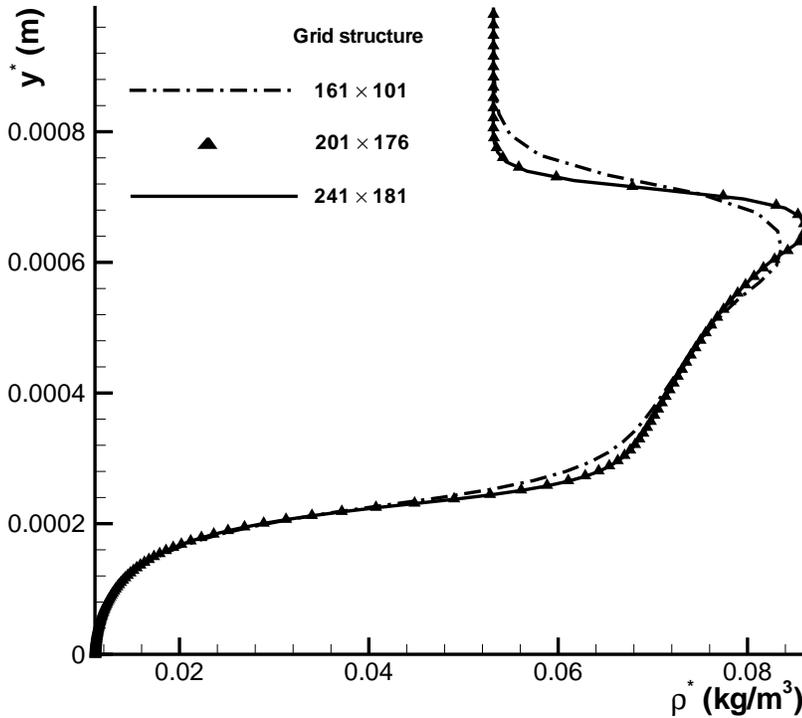


Figure 4: Comparison of density distributions in wall-normal direction at  $x^* = 0.0025$  m simulated by second-order TVD scheme based on three sets of grid structures.

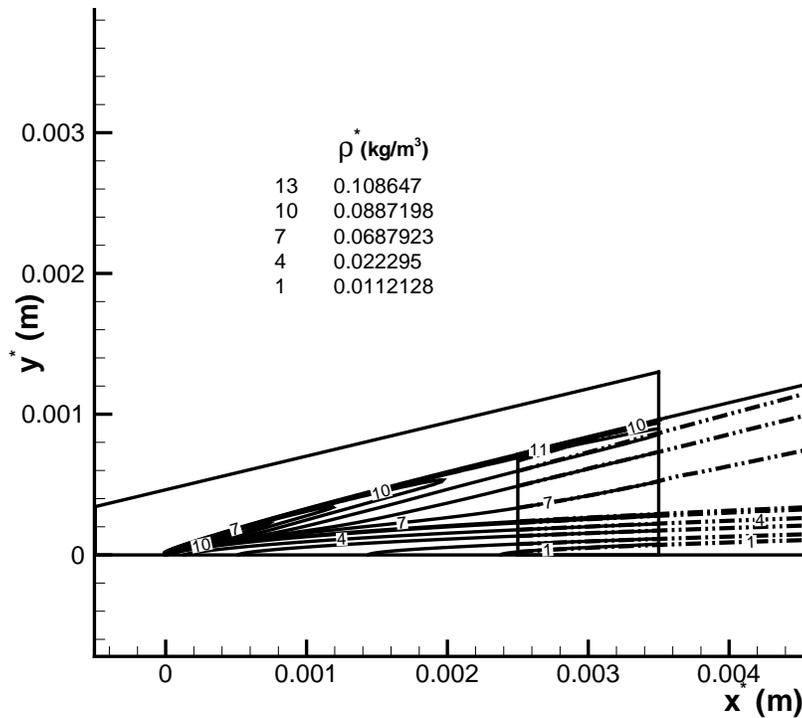


Figure 5: Density contours near the leading edge of the steady base flow obtained by the combination of second-order TVD scheme and fifth-order shock-fitting method.

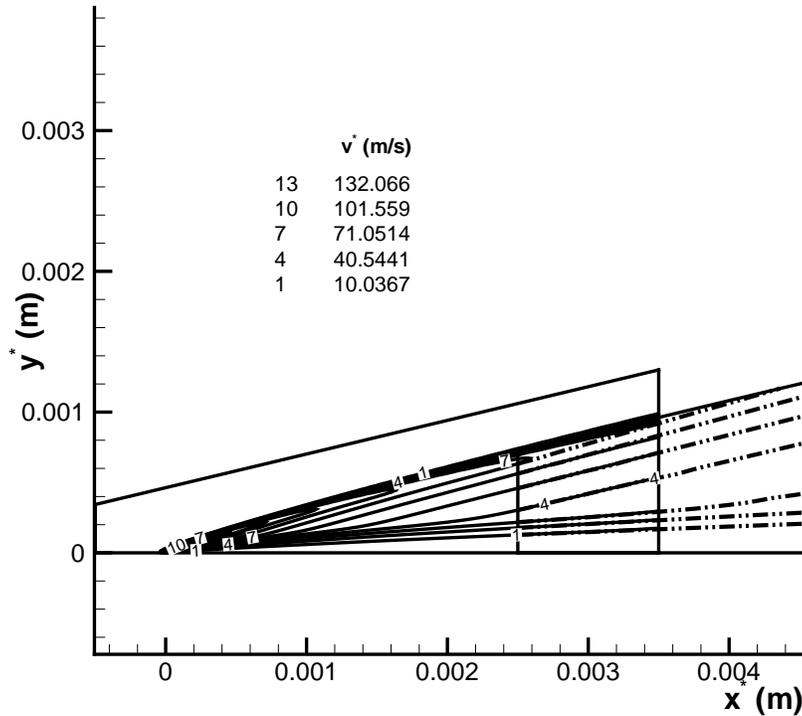


Figure 6: Wall-normal velocity contours near the leading edge of the steady base flow obtained by the combination of second-order TVD scheme and fifth-order shock-fitting method.

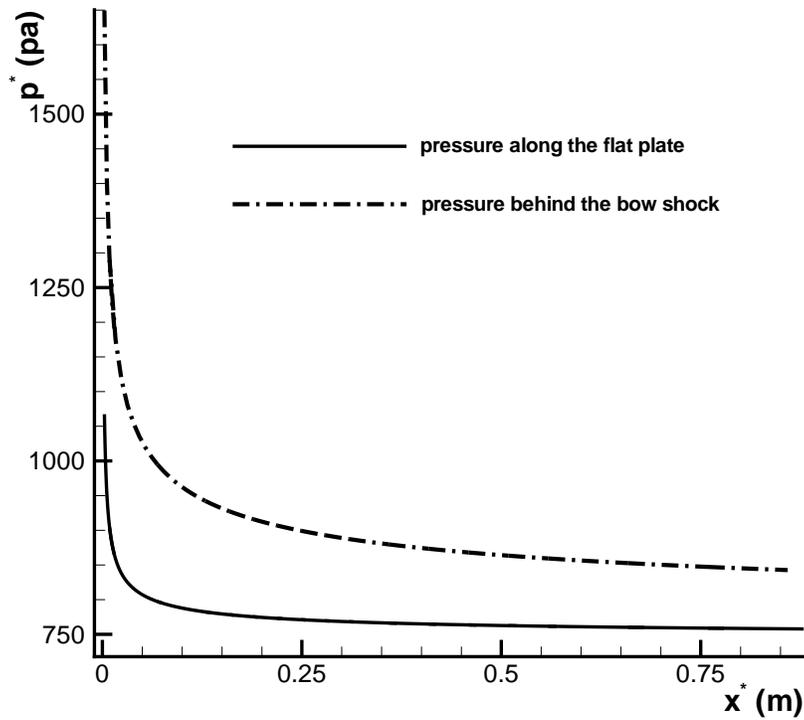


Figure 7: Pressure distributions along the flat plate and behind the shock of the steady base flow simulated by the fifth-order shock-fitting method.

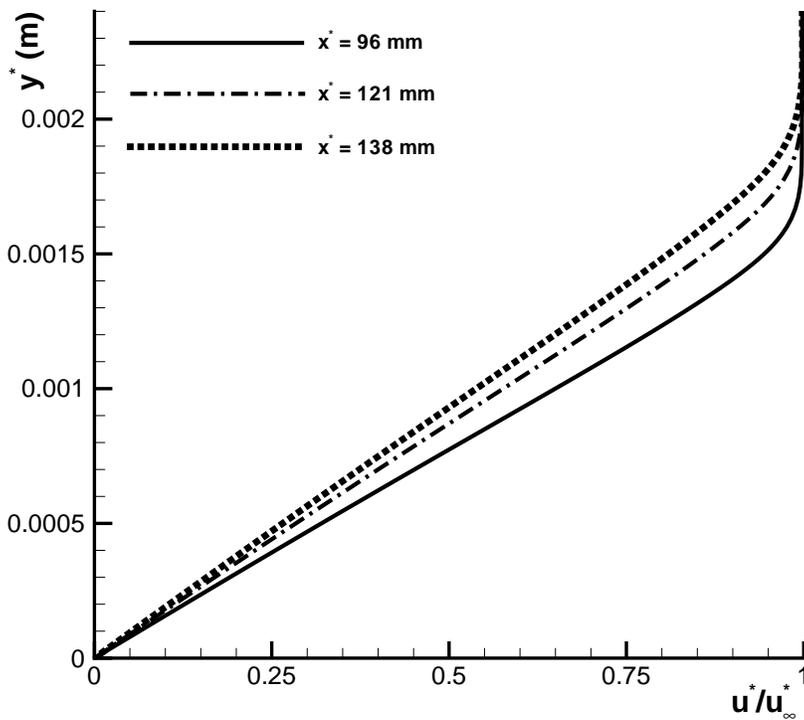


Figure 8: Distributions of dimensionless streamwise velocity across the boundary layer of the numerical steady base flow at three different locations.

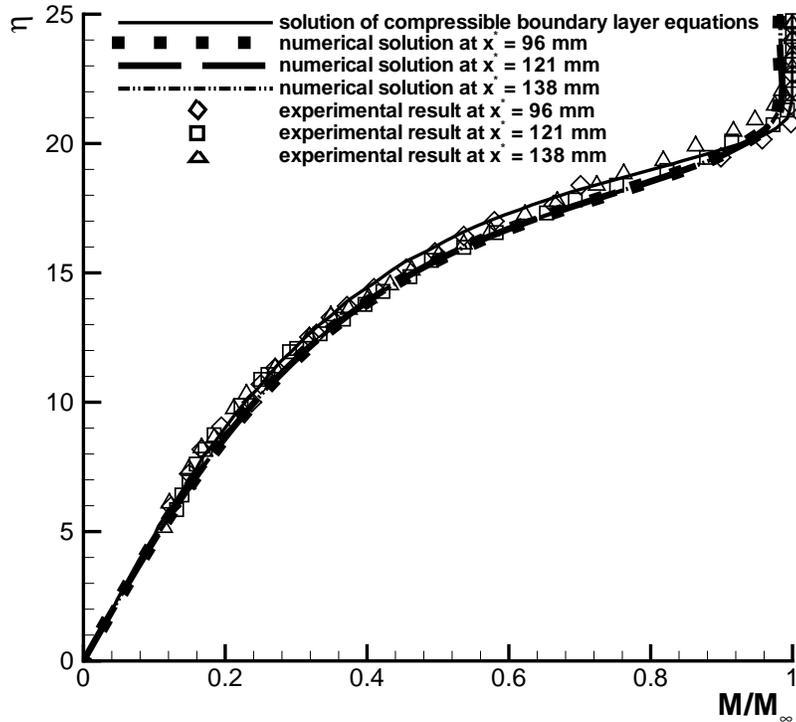


Figure 9: Comparison of normalized Mach number distributions across the boundary layer at three different locations.

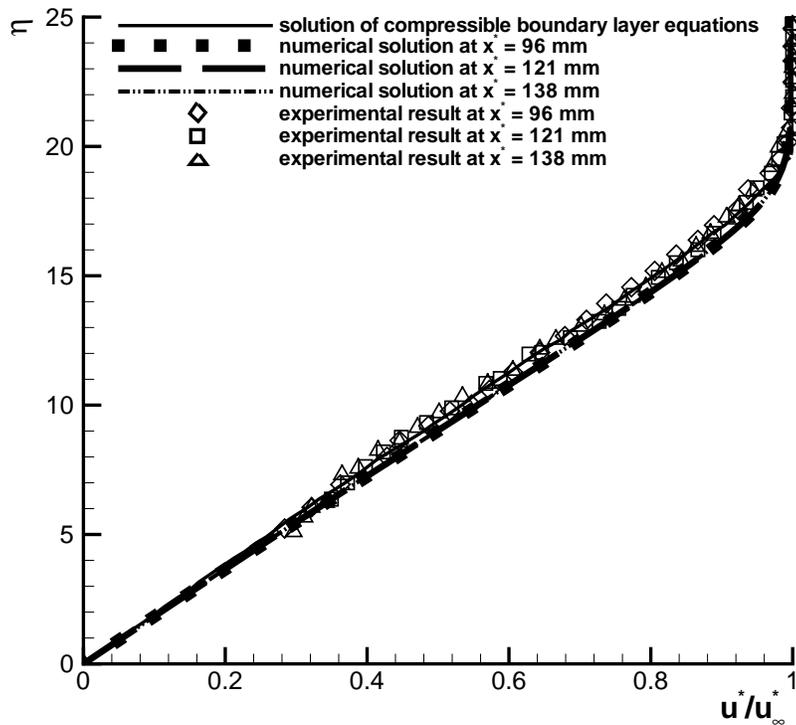


Figure 10: Comparison of dimensionless streamwise velocity distributions across the boundary layer at three different locations.

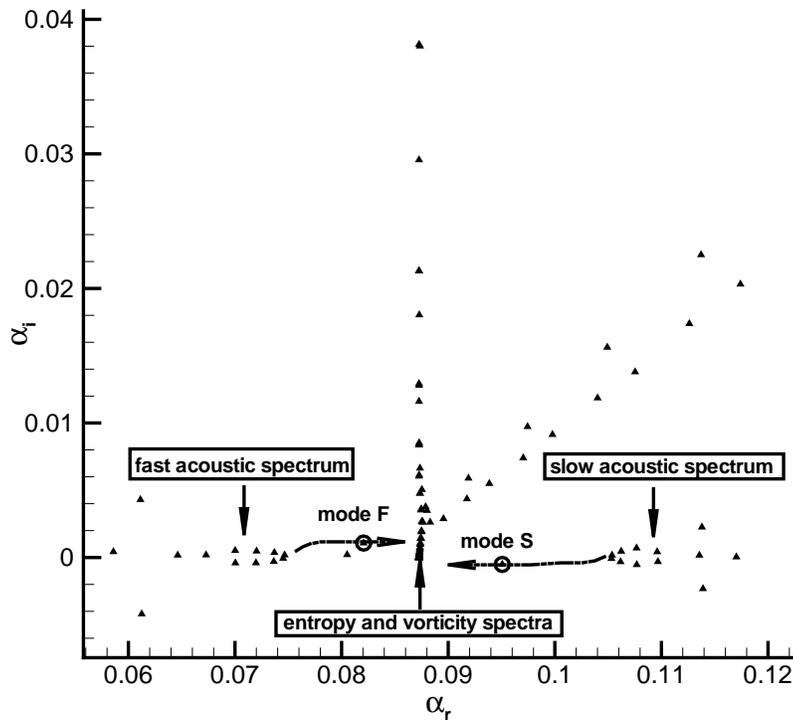


Figure 11: Eigenvalue spectra of the boundary-layer modes for a case at the frequency of  $f^* = 100$  kHz at  $x^* = 0.189$  m.

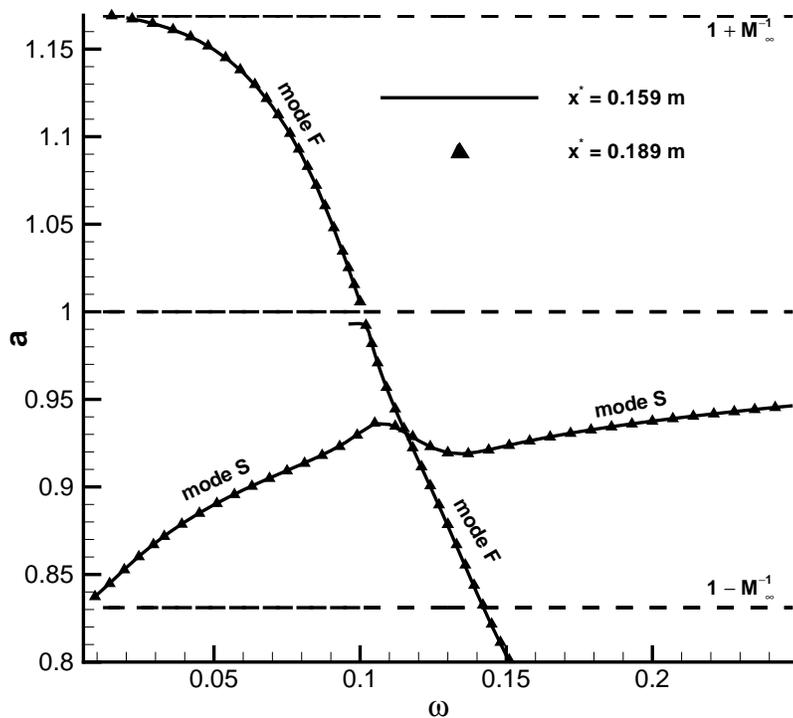


Figure 12: Distributions of dimensionless phase velocities of boundary-layer wave modes at two different locations versus dimensionless circular frequency.

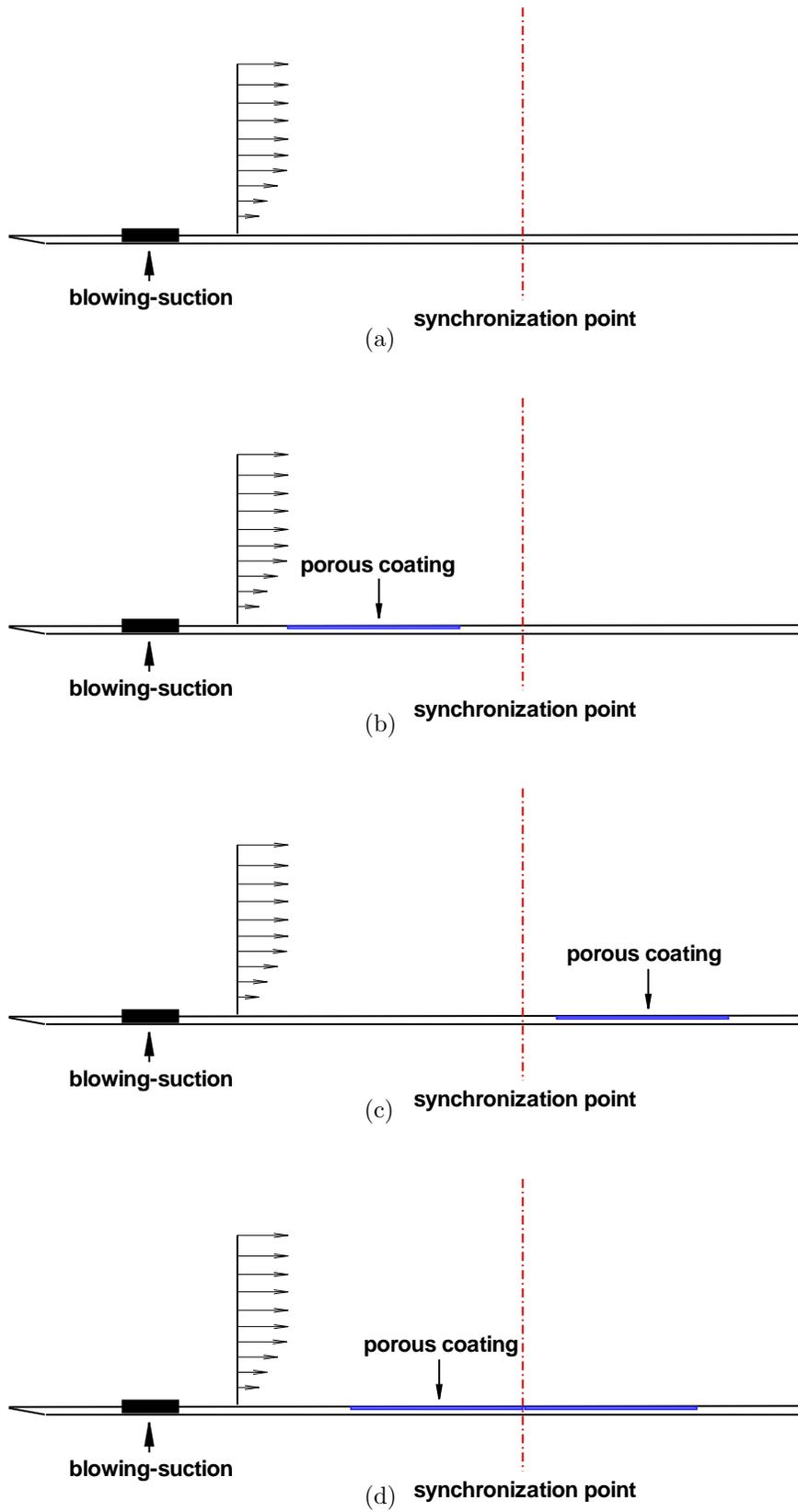


Figure 13: A schematic of porous coating locations and the synchronization point for the four cases: (a) case 1; (b) case 2; (c) case 3; (d) case 4.

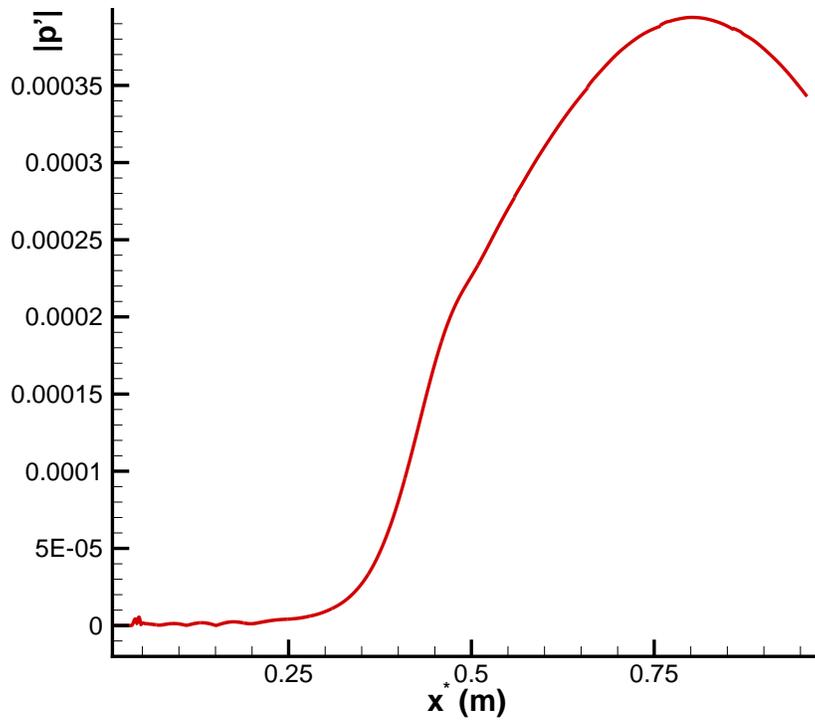


Figure 14: Amplitude distribution of pressure perturbation along the flat plate excited by the single-frequency blowing-suction at  $f^* = 100$  kHz.

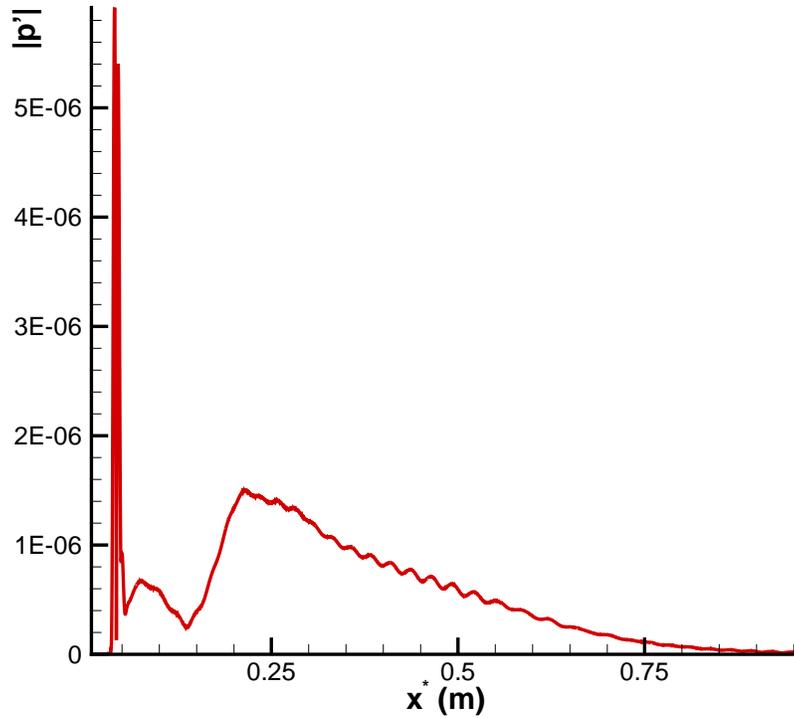


Figure 15: Amplitude distribution of pressure perturbation along the flat plate excited by the single-frequency blowing-suction at  $f^* = 150$  kHz.

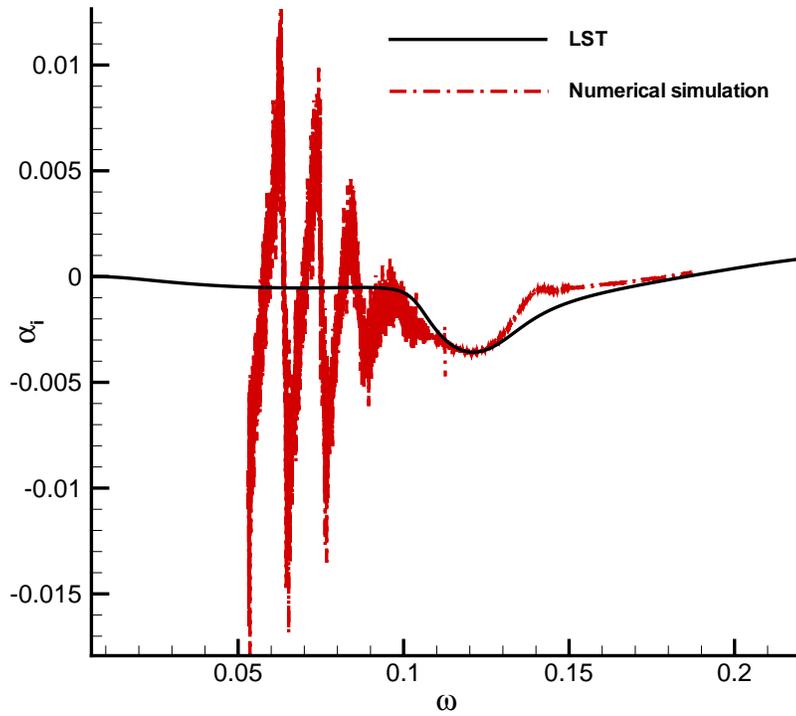


Figure 16: Comparison of numerical growth rate and LST growth rate of the perturbation at frequency of  $f^* = 100$  kHz.

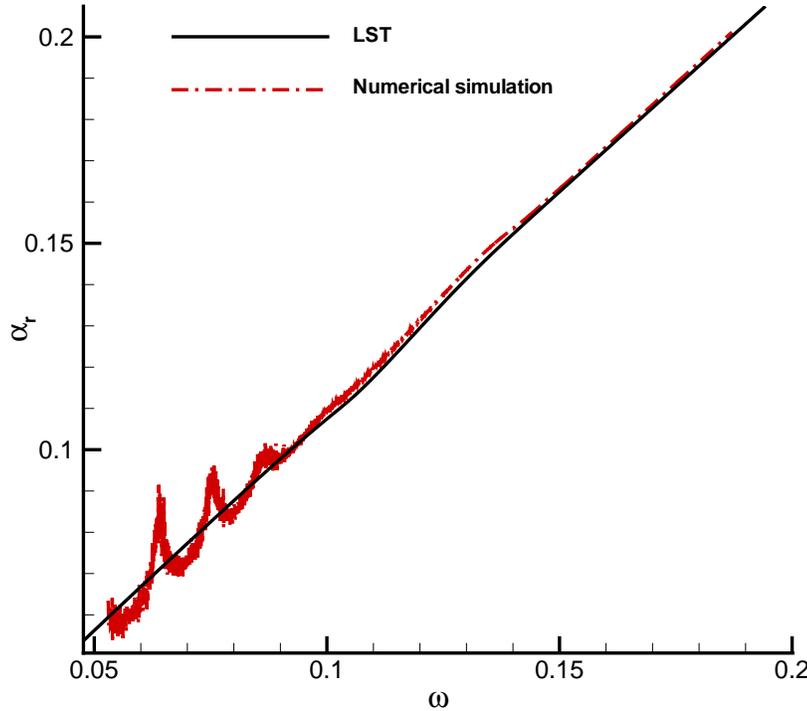


Figure 17: Comparison of numerical wave number and LST wave number of the perturbation at frequency of  $f^* = 100$  kHz.

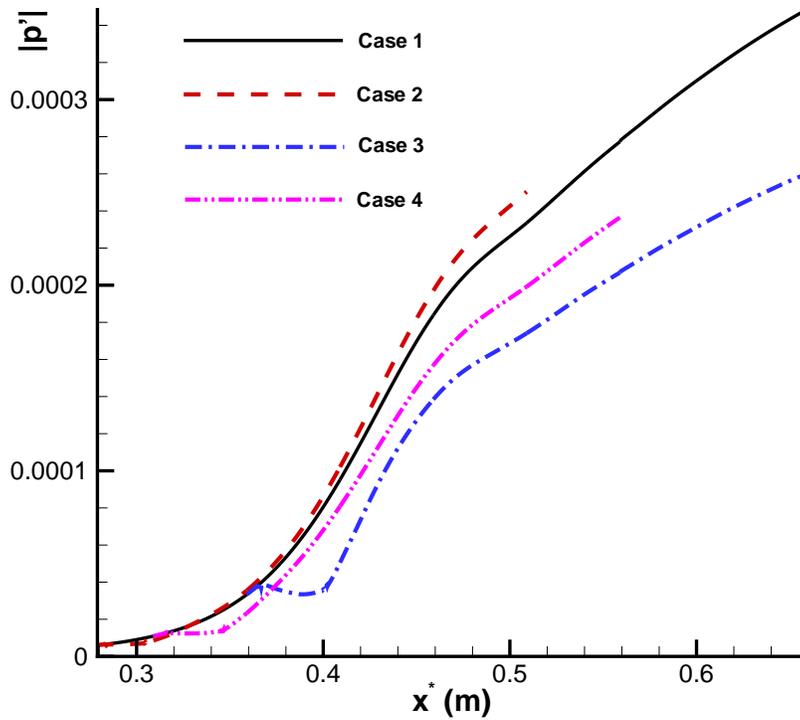


Figure 18: Amplitude distribution comparison of pressure perturbation along the flat plate for the four cases at frequency of  $f^* = 100$  kHz.

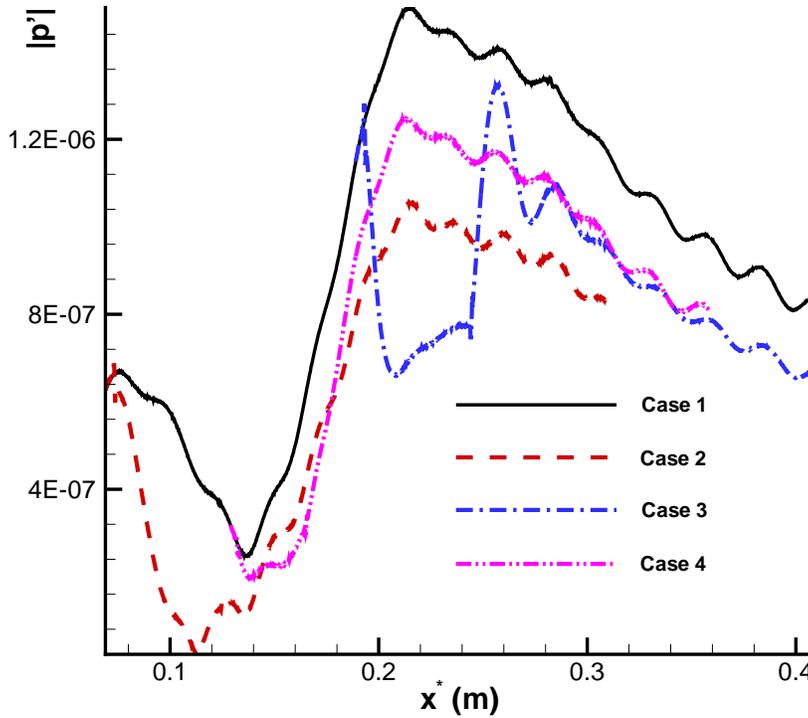


Figure 19: Amplitude distribution comparison of pressure perturbation along the flat plate for the four cases at frequency of  $f^* = 150$  kHz.